CHAPTER 6
THE MAIN SEQUENCE LIFE OF STARS

6.1 Introduction

In Chapter 4 we saw that if we determine the temperature and luminosity of a large number of stars and plot their positions on a Hertzsprung–Russell diagram, we will find that they are not randomly distributed. The majority lie on the main sequence, ranging from hot, luminous stars of spectral type O and B to faint, cool stars of spectral type M. The Sun (spectral type G2) is a very ordinary star lying near the middle of this sequence. The fact that we see so many stars on the main sequence leads us to believe that any star must spend a relatively large fraction of its lifetime there. The Sun's luminosity must have remained relatively constant for most of its life, otherwise the liquid water on the Earth (which is such an essential requirement for life) would have boiled away or frozen out, whereas the geological record tells us that oceans have been present for much of the Earth's lifetime of 4.5×10^9 years.

We have considered the basic processes of star formation in Chapter 5. All the evidence tells us that young stars evolve onto the main sequence, the most massive stars being the hottest and most luminous. In this chapter we will examine the physical properties and energy sources of stars on the main sequence. We can make use of our knowledge of the star that we know best, the Sun (Chapters 1 and 2), but it would be dangerous to use only the Sun as a model, as we would have no idea how typical of the whole population of stars it might be. It is therefore important to use observations of all stars, as described in Chapters 3 and 4, to try to piece together the life stories of different types of stars. When we understand more about the processes that dictate the structure of main sequence stars and produce their immense power output, we will be better placed to investigate how they evolve.

6.2 Stellar structure

6.2.1 Stellar models

We shall start our investigation of main sequence stars by looking at the physical conditions that exist inside a typical star and the way in which these parameters vary with position within a star. If we take our Sun as an example, you saw in Chapter 1 that these parameters can be measured for the outer layers, the corona, chromosphere and photosphere. How they change deeper in the Sun, however, can't be known from direct observation, as those regions are essentially inaccessible. We have to rely on indirect information such as the results of helioseismology (Section 2.2.6) which constrain theoretical models of the variation of temperature, pressure, density and composition with depth, as shown in Figures 2.1 and 2.2.

The situation with other stars is clearly much more problematic because results from asteroseismology provide less information since the discs of stars cannot be resolved. Yet, in order to understand the way that different stars evolve we *do* need to have an understanding of these interior conditions, because it is these that dictate, to a large extent, the nature, the rate and the extent of the nuclear processes which power the stars.

The study of these conditions involves the construction of a set of equations, known as the **equations of stellar structure**, which permit predictions of the various

physical properties (e.g. temperature, density, pressure) throughout a star. Although the equations are relatively simple, the associated theory, and the techniques for their solution are complex. However, by making various simplifying assumptions, it is possible to study basic properties of stars and to derive some limited quantitative information about their structure. We shall briefly look at the way in which some of the equations are derived, without in any sense attempting a rigorous determination.

Before that, however, we can make some general observations. Because energy appears to be lost from the surface of a star, in the form of radiation and perhaps particles, and is released within a star, it appears likely that the temperature generally increases with depth into a star. A temperature gradient in this direction is needed to maintain an outward flow of energy. Also, the pressure must increase with depth because of the increasing amount of overlying material.

■ Is the assertion that temperature and pressure increase with depth true for the Sun?

❑ Yes. Figure 2.1 shows that both parameters increase quite sharply with depth.

However, properties such as pressure, temperature, density, and various of the intrinsic properties of the material that constitute a star, such as composition, thermal conductivity, opacity (the degree to which the gas absorbs radiation), and the rate of energy generation, are all intricately interrelated, and it is one of the challenges of stellar physics to determine these relationships.

6.2.2 Internal temperatures and pressures

In order to set up the equations that describe the flow of energy, the pressure balance and other properties of a star, astronomers often resort to the strategy of considering a small volume at an arbitrary position within the star. They then write down the appropriate equations for that small volume only. By applying what are called boundary conditions (e.g. the fact that the temperature of a small volume at the edge of the star is simply the observed surface temperature) and other constraints (such as the fact that the masses of all small volumes added up is the total mass of the star), they look for consistent solutions to the equations that yield, amongst other things, profiles of temperature and pressure against depth in the star.

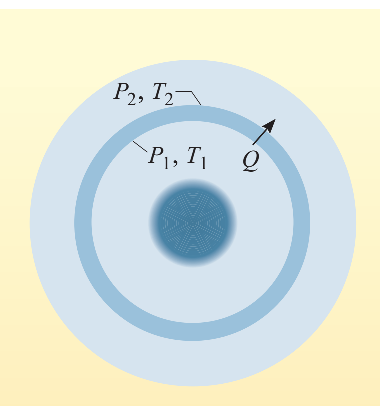


Figure 6.1 A thin spherical shell within a star to which calculations of various stellar properties are applied. The pressure and temperature at the inner and outer surfaces of the shell are calculated, as well as the heat flowing through it.

If we make the assumption that the star is spherically symmetric – in other words, that its properties vary in the same way in any radial direction away from the centre – then the small volume that we should consider is a thin spherical shell, as shown in Figure 6.1. In this figure, Q is the flux of energy flowing outwards in the radial direction through the shell. It has the units of energy per unit cross-sectional area perpendicular to the direction of flow per unit time. In order to maintain this flow of energy, there must be a temperature difference between the faces of the imaginary shell. Referring to Figure 6.1, T_1 must be greater than T_2 in order to maintain an outward flow of energy, i.e. we expect temperature to increase with depth.

The exact difference between T_1 and T_2 is related not only to the rate of energy flow (a higher rate of energy transfer generally requiring a larger temperature difference), but also to the pressure and composition of the gas itself.

One of the simplifying assumptions made is that the stellar material, at least for main sequence stars, behaves as a so-called perfect or **ideal gas**. If you have come across the concept of an ideal gas before, you may be surprised to hear that we can apply it at the high densities that are believed to exist deep inside a main sequence star. The

very high temperatures inside a star cause most of the gas to be ionized, and these ionized particles and electrons act as ideal particles. This means that we can use simple equations, such as the **ideal gas law**, which states, in one form:

$$P = k\rho T/m \tag{6.1}$$

where P is the gas pressure, ρ is the gas density, m is the average mass of the gas particles, k is the Boltzmann constant, and T is the gas temperature. Because m is dependent on the composition of the star, this form of the equation reinforces the fact that certain of the physical properties, such as pressure, do depend on composition.

How does the pressure vary with position throughout the star? To answer this question, we return to the imaginary shell in Figure 6.1. In a stable main sequence star, we assume that this shell will be stationary, i.e. there will not be significant net movement of mass on a large scale, so the forces acting on the shell must balance. Gravity acting on the shell will cause the material to be pulled towards the centre of the star. Opposing this will be the force due to the pressure in the gas. This balance is called **hydrostatic equilibrium**. If P_1 and P_2 are the pressures on the inner and outer faces of the shell, then as long as P_1 is greater than P_2, the pressure difference will result in a force acting outwards, opposing that due to gravity. The pressure in a body always increases with depth, because of the increasing mass of material above, and so P_1 must be greater than P_2. However, this gradient need not be sufficiently large to stop contraction. If there is an internal energy source generating a temperature gradient, then, as Equation 6.1 shows, this will tend to increase the pressure gradient. In a main sequence star the internal energy source is, of course, the nuclear power-house in the core. It enables the star to achieve stable equilibrium. The variation of pressure with depth can then in principle be calculated.

We have seen that a temperature difference across the shell maintains a flow of energy through that shell. If the shell is outside the region where energy is being released then the energy flowing into the shell is equal to that flowing out. However, if it is inside this region, then the rate at which energy flows out exceeds the rate at which it flows in by an amount equal to the rate of energy release within the shell.

The concepts described above can be used to define some of the equations of stellar structure. We will not attempt to derive any of these equations here as they can get quite complex and are beyond the scope of this book. However, we can, by making some very simple assumptions, make a surprisingly accurate estimate of the internal temperature of a star.

We consider a star, of mass M and radius R, composed of an ideal gas, as being divided into two hemispheres, each of mass $0.5M$, as indicated in Figure 6.2.

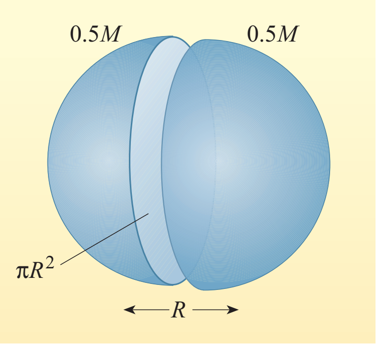


Figure 6.2 Schematic diagram of a star divided into two hemispheres for the purpose of estimating the central temperature.

The two hemispheres gravitationally attract each other with a force of magnitude given approximately by

$$\begin{aligned} F &= \frac{G(0.5M)(0.5M)}{R^2} \\ &= \frac{GM^2}{4R^2} \end{aligned}$$

We have assumed that the two hemispheres can be considered as point masses separated by a distance of R. Balancing this gravitational force on each hemisphere, which tends to make the star contract, is the outward pressure, P, of the hot gas. The pressure gradient here is provided by the difference between P inside the star,

and zero pressure outside it. Let us assume that P acts over the inner surface of the hemisphere, of surface area πR^2 (see Figure 6.2). So, remembering that pressure times area (i.e $P \times \pi R^2$) is force,

$$P\pi R^2 = \frac{GM^2}{4R^2} \tag{6.2}$$

We now need to use the ideal gas law:

$$P = k\rho T/m \tag{6.1}$$

The density, ρ, is equivalent to M/V, or Nm/V, where V is the volume containing N particles each of mass m. Equation 6.1 then becomes

$$P = \frac{kNmT}{mV} = \frac{NkT}{V} \tag{6.3}$$

For a sphere, $V = \frac{4}{3}\pi R^3$, so

$$P = \frac{3NkT}{4\pi R^3} \tag{6.4}$$

Thus, eliminating P from Equations 6.2 and 6.4,

$$\frac{3NkT}{4\pi R^3} \times \pi R^2 = \frac{GM^2}{4R^2}$$

which can be rearranged to give

$$T = \frac{GM}{3kR} \times \frac{M}{N}$$

The factor M/N represents the total mass of the star divided by the total number of particles – in other words it is the average mass of a particle, m_{av}, in the whole star. Thus we have the final result for the temperature of the interior of the star

$$T = \frac{Gm_{\text{av}}}{3k}\frac{M}{R} \tag{6.5}$$

where the constants are gathered into the first term. For an approximate calculation such as this one, we won't go far wrong if we assume that the star is composed entirely of ionized hydrogen, and so m_{av} is approximately equal to $\frac{1}{2}m_{\text{H}}$, where m_{H} is the mass of the hydrogen atom. (In an ionized gas a hydrogen atom of mass m_{H} consists of two separate particles, a proton and an electron.) If we substitute values appropriate for the Sun, we find $T \approx 4 \times 10^6$ K. We should expect this to be only an approximation to the true internal temperature, which will also vary considerably through the star, being highest in the core, and falling towards the edge. In the case of the Sun, more detailed calculations indicate a temperature in the core of 1.56×10^7 K. Our very simple calculation, therefore, which has made no assumption of the method of energy release, but has used only some simple laws of physics, has given the core temperature accurate to within a factor of four.

QUESTION 6.1

Identify some other gross assumptions that have been made in this derivation.

What this calculation has done is to show that the temperatures that are likely to be attained in a typical star are sufficiently high to trigger nuclear reactions. The types of nuclear reaction that occur are discussed in Section 6.3.

Equation 6.5 is also important because it indicates the way in which we might expect the interior temperature of a star to change for stars of different mass. From this equation we can write $T \propto M/R$. More realistic derivations show that this relationship applies to the *core* temperature of stars.

- How does the radius change if the star had double the mass? (You may safely assume that the mean density is roughly the same.)
- Mass $M = \frac{4}{3}\pi R^3 \rho$ so we can rewrite this as $R = (3M/4\pi\rho)^{1/3}$. If M is larger by a factor of 2 then R is larger by a factor of $2^{1/3} = 1.26$ or by about 26%.

The mass therefore increases more rapidly than the radius, causing the ratio M/R to be larger for stars of larger mass. It therefore follows that the temperature is predicted to rise with increasing stellar mass. This is very important and is at the root of the observation that luminosity increases dramatically for more massive main sequence stars – this **mass–luminosity relationship** was established in the 1920s by Eddington (Figure 6.3), on theoretical grounds. This is the relationship that, as we will show in Section 6.2.5, leads to the result that the lifetime of massive stars is shorter than the lifetime of less massive ones.

ARTHUR STANLEY EDDINGTON (1882–1944)

Arthur Eddington (Figure 6.3) won a scholarship and started university in Manchester a few months before his 16th birthday. He got a first class degree in physics and a scholarship to Trinity College Cambridge at age 19. He completed the three-year maths course at Cambridge in two years coming top of the final year class as well – the first time a second-year student had done that! In 1907 he won a Trinity College Fellowship, on the basis of a five-page thesis, and at the age of 30 became Plumian Professor.

During the First World War he received, via neutral Netherlands, a paper by Einstein describing relativity theory. One of the first people to recognize the importance of this theory, Eddington also recognized that the predicted gravitational bending of light could be checked observationally during a solar eclipse. He led an eclipse expedition in 1919 which confirmed the theory.
He is also known for his significant work in stellar dynamics and stellar structure, and for the disagreements he had with Chandrasekhar (Figure 9.2) and Jeans (Figure 5.1)! He had excellent physical insight; as early as 1917 he was speculating that the energy source in stars was sub-atomic, involving the transmutation of hydrogen into other elements. He discovered the fundamental role that radiation pressure plays in maintaining stellar equilibrium, argued the importance of radiative energy transfer in stars and showed that a star's equilibrium was maintained by the balance between gravity and the outward forces of gas pressure and radiation pressure. His announcement in 1924 that the luminosity of a star depends almost entirely on its mass revolutionized ideas about stellar evolution. Eddington was an accomplished writer, introducing many to the science of astronomy and was considered one of the greatest astronomers of his time. He was elected a Fellow of the Royal Society in 1914 and knighted in 1938.

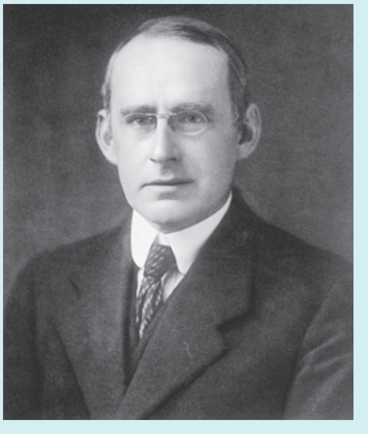


Figure 6.3 Arthur Stanley Eddington. (Royal Astronomical Society)

6.2.3 Energy release and transport

If we accept that nuclear processes (which differ depending on a star's mass) are responsible for energy release within main sequence stars, where do these processes take place within a star? It has to be stressed that we have essentially no *direct* information on this question, since we cannot see directly into the interiors of stars (although we can probe the interior of the Sun and other stars using helioseismology/asteroseismology – see Section 2.2.6). It is possible, however, to use the results of modelling of the interiors of stars through the equations of stellar structure, together with knowledge of the relevant nuclear processes, to answer this question.

■ Why can't we assume that nuclear processes are occurring throughout the volume of most main sequence stars?

❑ The surface temperatures of stars are too low for nuclear processes to be taking place at the surface, so clearly these reactions do not take place throughout the whole star.

For stars of all masses, nuclear reactions take place where the temperatures are greatest, which is at the centre and in a surrounding region, called the core. Because the nuclear reaction rates vary so much with temperature, this means that, moving out from the centre of a star, the boundary defining the limit of nuclear reactions is fairly sharp. The size of this region varies with the mass of the star, as do the types of reaction that predominate, and the mechanism by which energy is transported to the outer layers of a star.

■ By what mechanisms can energy be transported from one place to another, and which do you think play an important part in main sequence stars?

❑ There are three mechanisms – conduction, convection and radiation. If we regard the Sun as typical of main sequence stars, then we would expect, as we saw in Section 2.2.5, that convection and radiation are the dominant mechanisms of energy transfer in these stars.

Let's consider stars of low mass (less than $\sim 1.5M_{\odot}$) on the main sequence (called **lower main sequence stars**). For stars with masses $\sim 1M_{\odot}$ the rate of energy release does not increase sufficiently towards the centre to set up the temperature gradient necessary to initiate convection in the core. In other words, the core is non-convective. Immediately outside the core, the temperature gradient is still too small to cause convection, and so radiation is the primary mechanism by which energy is transferred. Further out, however, there is a region in which convection does take place, in the form of a convective zone or envelope. This convective envelope starts deeper in stars of lower mass, and for masses less than $\sim 0.3M_{\odot}$ the convective envelope reaches right down to the centre.

If the mass of a star exceeds about $1.5M_{\odot}$ (termed **upper main sequence stars**), the temperature of the core is sufficiently high for a different set of nuclear reactions (we'll be looking at these shortly) to become predominant. The energy release in the core is then sufficiently concentrated to trigger convective instability and the centre of the star is convective. In fact, this convective zone may extend beyond the core in which nuclear reactions are taking place. The situation for both upper and lower main sequence stars is shown in Figure 6.4.

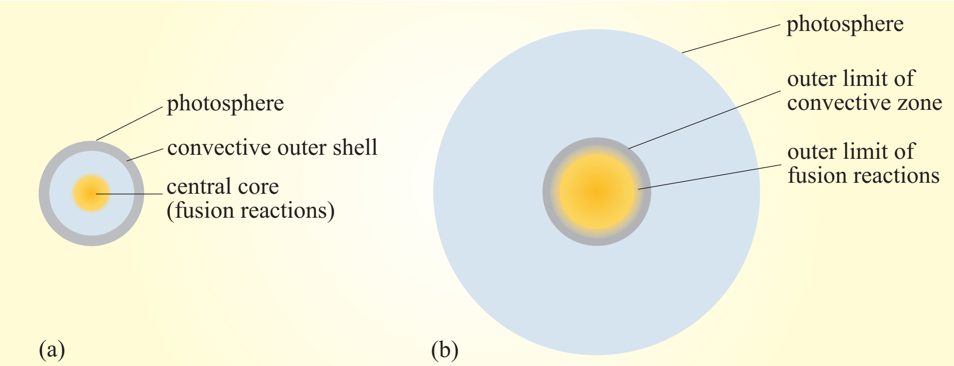


Figure 6.4 Schematic cross-section of a star of (a) $1M_\odot$ and (b) $5M_\odot$, showing the difference between the location of the convective regions.

Figure 6.5 illustrates how the situation varies with stars of different mass. It shows the extent of the **convective core** or the **convective envelope** as a function of the mass of a star. The vertical axis represents the fraction of the total mass occupied by each region.

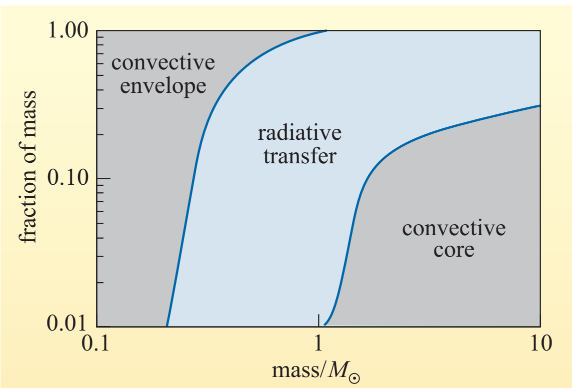


Figure 6.5 The fraction of the mass of a star in the convective core and envelope as a function of stellar mass. Note that the mass is concentrated towards the centre of the star so the convective envelope of a $1M_\odot$ star (see Figure 6.4) contains only a small fraction of the mass.

QUESTION 6.2

In a star of $\sim 1M_\odot$, will the composition of the core change during its main sequence lifetime? If so, how?

6.2.4 Why is there a main sequence?

In trying to answer the question 'Why is there a main sequence?' we need to consider why stars have particular values of luminosity and temperature and why these values remain constant for so long.

Let's first consider the **stability** of stars on the main sequence. You have already seen that a stable star is in hydrostatic equilibrium, i.e. the inward force of gravity on each layer of a star is balanced by the net outward pressure forces.

■ What would you expect to happen to the pressure of the gas in the star if the whole star is 'squashed' a small amount, i.e. its radius is decreased at constant temperature?

❑ The density of the gas in each layer of the star would rise because the volume has decreased. Since $P = k\rho T/m$ (Equation 6.1) for each layer of the star, the pressure will also rise.

Thus the net outward force due to the variation of pressure with depth will also rise and there will no longer be a balance with the inward gravitational force. The star will therefore expand back to its original dimensions. (This is a simplification since the gravitational force will also rise a small amount if the star is 'squashed', but this will be accompanied by a rise in temperature, due to conversion of the gravitational potential energy into kinetic energy of the gas particles, and hence a rise in pressure.) This means that a star on the main sequence is very stable against any dimensional changes. (We will consider the longevity of most stars on the main sequence in the next section.)

Now we will briefly consider the fundamental matter of why there is a main sequence in the first place! In 1926, Henry Norris Russell (Figure 4.2b) and the German physicist Heinrich Vogt (pronounced 'voit') (1890–1968) derived a result, sometimes called the **Russell–Vogt theorem**, which can be stated as: 'The equilibrium structure of an ordinary star is determined uniquely by its mass and chemical composition'. To put it another way, we can say that a certain mass of stellar material of fixed composition can reach only *one stable configuration*. This stable configuration will correspond to one point on the H–R diagram. A star of different mass occupies a different point on the H–R diagram. This is exactly what we see along the main sequence, as in Figure 4.8. Thus, we have a main sequence because the stars on it are stable, with similar chemical compositions, but with different masses.

How can we reconcile the Russell–Vogt theorem with the fact that the H–R diagram, apart from the main sequence, also contains other regions, such as those of red giants, supergiants and white dwarfs? The existence of these regions would seem at first to undermine the theorem. But, as we shall see later, their existence is not contradictory, for it is the chemical composition of these stars that is different, and this allows them to occupy different regions on the H–R diagram.

6.2.5 Main sequence lifetimes

Consider the duration of a star's life on the main sequence.

■ You have already come across some evidence in Chapter 4 that gives a clue as to how a star's lifetime on the main sequence varies with the star's properties. Can you remember the source of that evidence and what it tells us?

❑ The evidence comes from looking at the H–R diagram for star clusters (Section 4.2.5). These show that the more massive stars have shorter main sequence lifetimes.

Let's try to be slightly more quantitative about the matter of main sequence lifetime. The lifetime is given by the energy available divided by the luminosity. Masses and luminosities derived from observations (see Sections 3.3.7 and 3.3.3 respectively) imply that the main sequence luminosity depends on a high power of the mass. On average, $L \propto M^4$ (although the actual power varies between 3 and 5 depending on the mass, the exact value does not affect the basis of the discussion below). If the mass of one star is twice the mass of another, the luminosity of the first is 2^4 or 16 times more than the second. The reason for this sensitive dependence on mass is the temperature of the core, which has important consequences for the star because the relevant nuclear reaction rates are *very* sensitive to temperature.

In order to estimate the lifetime of a star on the main sequence, we shall make the assumption that the energy available is proportional to the total mass. Therefore, the main sequence lifetime is given by

$$t \propto \frac{\text{energy supply}}{\text{rate of release}} \propto \frac{M}{L} \propto \frac{M}{M^4}$$

and so $\quad t \propto M^{-3} \qquad (6.6)$

This tells us that the lifetimes of stars on the main sequence decrease ever more rapidly for stars of higher mass. We now have an approximate relationship for the way in which the main sequence lifetime of a star depends on its mass – but the above relationship is not an equality but a proportionality. For example, a relationship like $t \propto M^{-3}$ tells us that for a star of double the mass, its main sequence lifetime is a factor of $(2)^{-3}$ or $\frac{1}{8}$ as long. In other words, the lifetime is one-eighth of that of the less massive star. However, we still need to fix, or 'calibrate', the lifetime scale. This can accomplished by a variety of means, usually consisting of a mixture of observational and theoretical approaches. Using the Sun for calibration gives a main sequence lifetime of about 1×10^{10} years for a mass of $1M_\odot$.

More sophisticated calculations give a dependence of main sequence lifetime on mass as shown in Table 6.1. The five most massive stars listed are upper main sequence, the remaining being lower main sequence stars.

It follows from Table 6.1 that massive stars, such as those of $15M_\odot$, are predicted to have relatively short main sequence lifetimes, perhaps as brief as 10 million years. This means that many of the massive upper main sequence stars currently observed must have been formed fairly recently on the astronomical timescale, and therefore provide further evidence that new stars are being formed even today.

Table 6.1 Selected properties of main sequence stars of various masses.

Mass/$M_\odot$	Luminosity/$L_\odot$	Surface temperature/K	Main sequence lifetime/yr
0.50	0.03	3800	2×10^{11}
0.75	0.3	5000	3×10^{10}
1.0	1	6000	1×10^{10}
1.5	5	7000	2×10^{9}
3	60	11000	2×10^{8}
5	600	17000	7×10^{7}
9	4000	23000	2×10^{7}
15	17000	28000	1×10^{7}
25	80000	35000	7×10^{6}

6.3 Nuclear reactions

Before we look at some of the details of nuclear fusion in main sequence stars, we should ask whether there is any other possible way in which the luminosity of these stars could be explained. In the following two questions you are asked to consider two particular alternative energy sources and to work out for how long each could keep the Sun shining at its present rate.

QUESTION 6.3

Suppose that it is chemical energy that is responsible for the luminosity of the Sun. Assuming that 1 kg of material in the Sun yields an energy output of 3.5×10^7 J (a value typical for coal-burning on Earth), determine for how long this mechanism could sustain the Sun's present luminosity.

QUESTION 6.4

Assuming that a star like the Sun contracts to a tenth of its present size, and in doing so releases gravitational energy, for how long could this power the Sun at its present luminosity? (You can assume that the gravitational energy of a sphere of radius R and mass M is approximately $-GM^2/R$.)

The answers to Questions 6.3 and 6.4 show that these two methods of releasing energy give solar lifetimes that are short compared to geological estimates of the lifetime of the Earth. It was not until the development of ideas on the structure of matter and on nuclear processes, in the early and middle part of the 20th century, that it was eventually appreciated that nuclear fusion could provide the vast amount of energy needed to power most stars. In fact, it appears to be the only possible energy source that we know of that will do the job.

In the study of the Sun and its power source in Chapter 2, you were introduced to some basic ideas concerning atoms, nuclei and nuclear processes. We shall call heavily on some of these ideas and concepts in our study of the evolution of stars both in this and subsequent chapters. However, if you need to know more about nuclear processes, this is an appropriate time to study Box 6.1.

BOX 6.1 FUSION REACTIONS AND ENERGY RELEASE

In Section 2.2.4 you met Einstein's famous equation, namely $E = mc^2$, which links mass and energy. When applied to a nucleus, of mass m, it enables us to determine the *rest energy*, mc^2, of any nucleus. This can be thought of as the energy that would be released if the nucleons (protons and neutrons) were annihilated and converted into energy alone. It is possible for us to measure the rest energy of most nuclides to high accuracy. Figure 6.6 shows a plot of the rest energy *per nucleon*, mc^2/A, as a function of the mass number A, the number of nucleons in the nucleus. The general property of the curve is that the rest energy per nucleon initially decreases fairly rapidly with increasing mass number; there is then a broad minimum around values of A between 50 and 60. Then the rest energy per nucleon gradually increases for nuclei with higher values of A. Note that ${}^{56}_{26}\mathrm{Fe}$ is a nuclide with one of the *lowest* rest energies per nucleon.

We can use Figure 6.6, or more detailed tabulated data, to work out whether a reaction is **exothermic** or **endothermic** – in other words, does the reaction release energy or does it require an input of energy in order for it to take place?

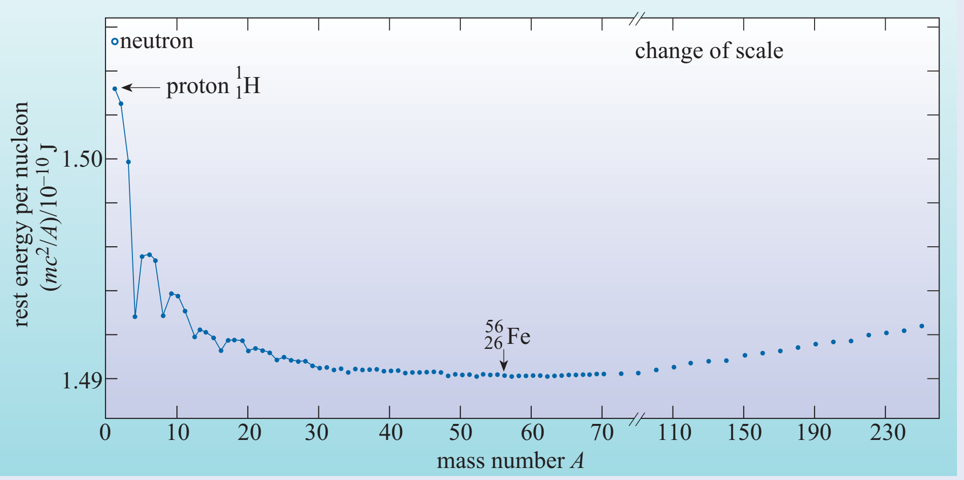


Figure 6.6 The variation of rest energy per nucleon as a function of mass number. At each mass number, the nuclide with the lowest rest energy per nucleon is shown. Beyond $A = 70$, only some mass numbers are shown.

■ Without *calculating* rest energies, use Figure 6.6 to decide whether the reaction

$$^{12}_{6}\text{C} + {}^{4}_{2}\text{He} \rightarrow {}^{16}_{8}\text{O} + \text{photon}$$

is exothermic or endothermic. (At each of the mass numbers 4, 12 and 16, these are the nuclides with the lowest rest energy per nucleon.)

❑ Figure 6.6 shows that the rest energy *per nucleon* is *lower* in the product $^{16}_{8}$O than in either of the reactants $^{12}_{6}$C or $^{4}_{2}$He. With the same number of nucleons (16) in the product as in the reactants, it follows that the sum of the rest energies of the reactants exceeds that of the product, and so the reaction *releases* energy, i.e. it is exothermic. (The fact that the number of nucleons in the reactants equals that in the product is in accord with the law of conservation of baryon number.)

To find the energy released in the reaction we make a quantitative calculation.

■ Calculate the energy released in the reaction. (The rest energies for the carbon, helium and oxygen nuclei are 1.7904×10^{-9} J, 5.9720×10^{-10} J and 2.3865×10^{-9} J, respectively.)

❑ The sum of the rest energies of the reactants on the left-hand side of the reaction is 2.3876×10^{-9} J. For the product, on the right-hand side, the rest energy is 2.3865×10^{-9} J. There is thus an excess of 1.1×10^{-12} J in the reactants.

The translational kinetic energy of the reactants is not very different from that of the product, and so nearly all of the energy released goes into the photon.

■ What type of photon is this?

❑ Using Equation 1.3, ($\varepsilon = hf$) we find

$$f = \varepsilon/h = (1.1 \times 10^{-12}\ \text{J})/(6.6 \times 10^{-34}\ \text{J s}) = 1.7 \times 10^{21}\ \text{Hz}$$

We can see from Figure 1.36 that this frequency corresponds to a γ-ray photon.

The fusion of two light nuclei to form a single nucleus with $A < 56$ is exothermic for nearly all pairs of reactants. However, can we determine which nuclear reactions are fast enough to be significant in the interior of stars? Various factors contribute to the rate of a particular nuclear reaction. We shall look at those factors in a qualitative way only.

First, we would clearly expect the rate to depend on the concentration (the number density) of the reactants. For the simplest case of a reaction between pairs of particles, the rate, and thus the energy release per unit volume, is proportional to the *product* of the concentrations of the two interacting types of particles.

Another crucial factor is the ability of two nuclei to get close enough to each other to react.

■ What normally prevents two nuclei from approaching very close to each other?

❑ As nuclei are positively charged, it is the repulsive electrical force that tends to keep them apart.

The electric charge on a nucleus is Ze, where Z is the atomic number of the nucleus, and e is the charge on the proton (1.6×10^{-19} C). The *magnitude*, F_e, of the repulsive electrical force between nuclei with atomic numbers Z_1 and Z_2 is given by

$$F_e = A_0(Z_1e)(Z_2e)/r^2 \qquad (6.7)$$

and the electrical potential energy is given by

$$E_e = A_0(Z_1 e)(Z_2 e)/r \qquad (6.8)$$

where $A_0 = 8.99 \times 10^9$ N m^2 C^{-2} is a universal constant, the Coulomb constant, and r is the separation of the nuclei. (Note how similar these equations are to those for gravitational force, Equation 3.17, which is also proportional to the inverse square of the distance separating two point masses.) Nuclear reactions can occur if the particles approach close enough to each other in spite of the repulsive electrical force; this occurs if the relative velocity of the two particles is high enough. Of course, the temperature is the main factor that dictates the relative velocity of particles in a gas. Although for a given temperature we can calculate the *average* velocity of a particle from its translational kinetic energy $E_k = 3kT/2$ (Equation 4.2), the particles actually exhibit a distribution of velocities with some lower and some higher than the average. However, it turns out that at the temperatures typical of the interiors of stars, the probability of one nucleus having sufficient energy to overcome the repulsive electrical force between nuclei is vanishingly small (see Question 6.5). So how then does nuclear fusion occur in stars? The answer called 'quantum mechanical tunnelling' lies in the details of quantum mechanics and the wave-like properties of atomic nuclei. The result is that for a given nucleus there is a small but non-zero probability that it can approach closely enough to another nucleus for nuclear fusion to occur despite the repulsive forces. As the temperature rises this probability increases.

QUESTION 6.5

Use Equation 6.8 to find the electrical potential energy of two hydrogen nuclei ($Z = 1$) at a separation of $r = 10^{-15}$ m (this is the distance at which the forces which hold nuclei together dominate and the two nuclei can combine). Use Equation 4.2 to determine the *average* thermal kinetic energy of a particle in the core of the Sun (assume $T = 1.6 \times 10^7$ K) to show that the temperature is too low for such particles to overcome the electrostatic forces.

We can also see from Equation 6.7 that the higher the electric charges of interacting nuclei, the greater is the repulsive electrical force between them. This generally means that nuclear reactions between light elements (which contain a small number of positively charged protons) occur at an appreciably faster rate at lower temperatures than reactions between heavy elements (which contain a large number of protons).

With this information, you should begin to see that light elements in a star can be gradually converted into heavier elements as a star evolves, and that, if the temperature within the star rises, these heavier elements can also undergo nuclear reactions. Figure 6.6 shows that most of these fusion reactions are exothermic until the product nucleus has a mass number of about 56.

6.3.1 The pp chain

You have already seen in Section 3.3.6 that the majority of stars have a remarkably similar composition to each other and to the Sun: 92% of all nuclei are hydrogen, about 7.8% are helium and the remaining 0.2% is made up of the other elements. It is clear that we should be looking to hydrogen for the source of nuclei to take part in the exothermic nuclear fusion reactions described in Box 6.1. The most important series of nuclear reactions occurring in main sequence stars are those that convert hydrogen into helium. This is termed **hydrogen burning** (although of course, the term here has nothing to do with setting fire to anything!).

There are various routes by which hydrogen can be converted into helium. However, some reactions are excluded by the conservation laws.

■ Can you remember which conservation laws are relevant to nuclear reactions?

❑ They are the conservation of charge, baryon number and energy.

The hydrogen burning routes that are not excluded by the conservation laws each involve a chain of reactions, whose overall effect is the conversion of hydrogen nuclei into helium nuclei. Several such chains are believed to play an important part

in the cores of main sequence stars. The particular temperature in the Sun's core dictates that the ppI chain (Section 2.2.4) should dominate, but that isn't necessarily the case in other stars. In stars with cores of progressively higher temperature than the Sun, we find that two other chains become important. These two are called the **ppII** and the **ppIII chains**.

- ■ What do the names ppII and ppIII tell you about these particular nuclear reaction chains?
- ❑ By analogy with the ppI chain, these reaction chains start with the interaction of two protons.

Moreover, for each of the pp chains, the net effect is the production of a helium nucleus, ${}^{4}_{2}\mathrm{He}$, from four protons (${}^{1}_{1}\mathrm{H}$) (Figure 6.7).

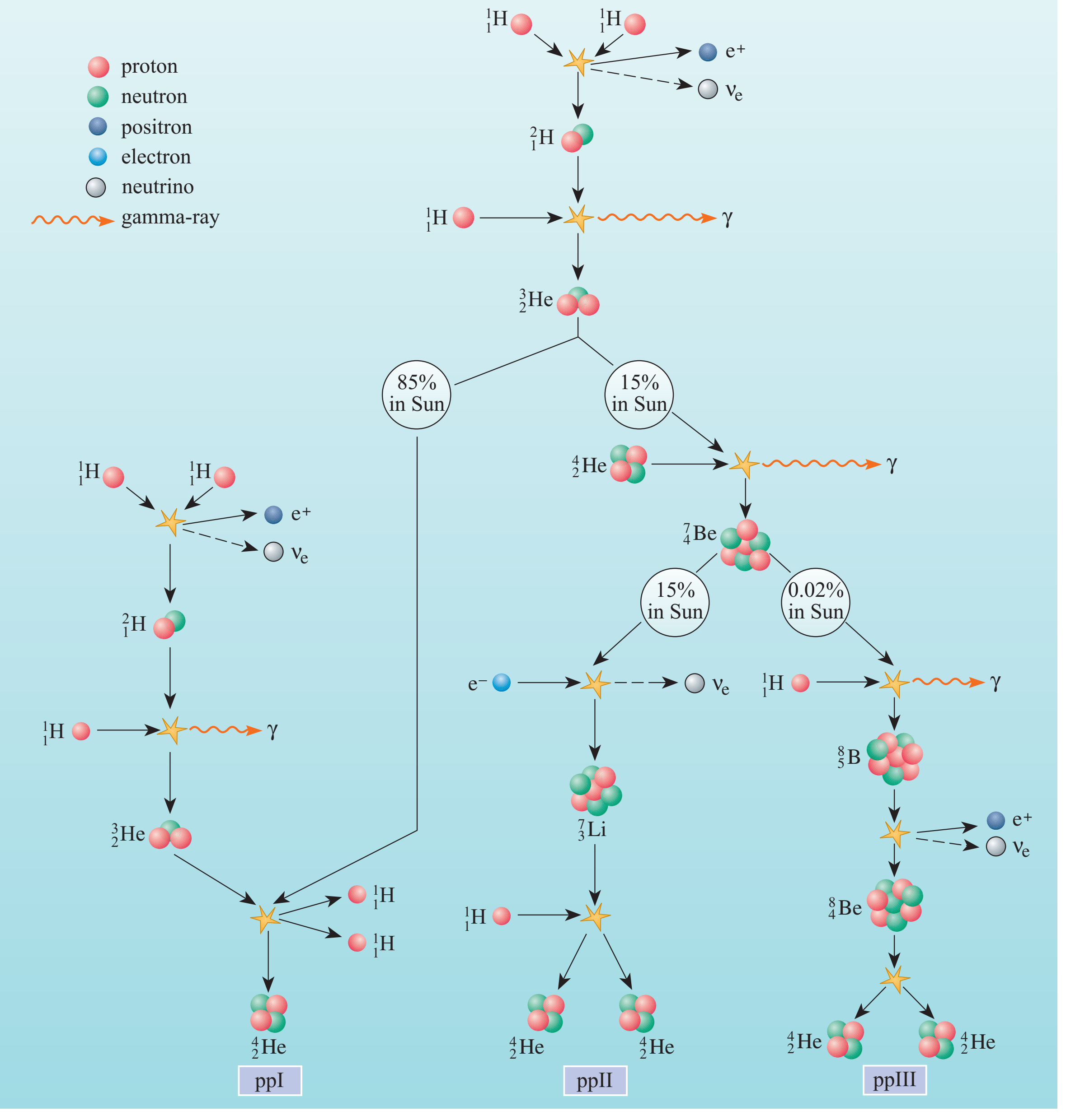


Figure 6.7 Details of each of the processes which convert hydrogen into helium in stars in the pp chains.

The neutrinos produced in the pp chains, by virtue of their very low probability of interacting with other matter, essentially all escape from the star in which they are produced. They carry off about 2% of the energy released in the formation of every helium nucleus (see Section 2.2.6 for details of the detection of neutrinos from the Sun).

6.3.2 CNO cycle

As we move to stars with yet hotter cores, we find a different set of reactions becoming important. These are reactions that involve the nuclei of carbon, nitrogen and oxygen. As in the pp chains, the net effect of this set of reactions is the production of a $^{4}_{2}\mathrm{He}$ nucleus from four protons. However C, N and O act as

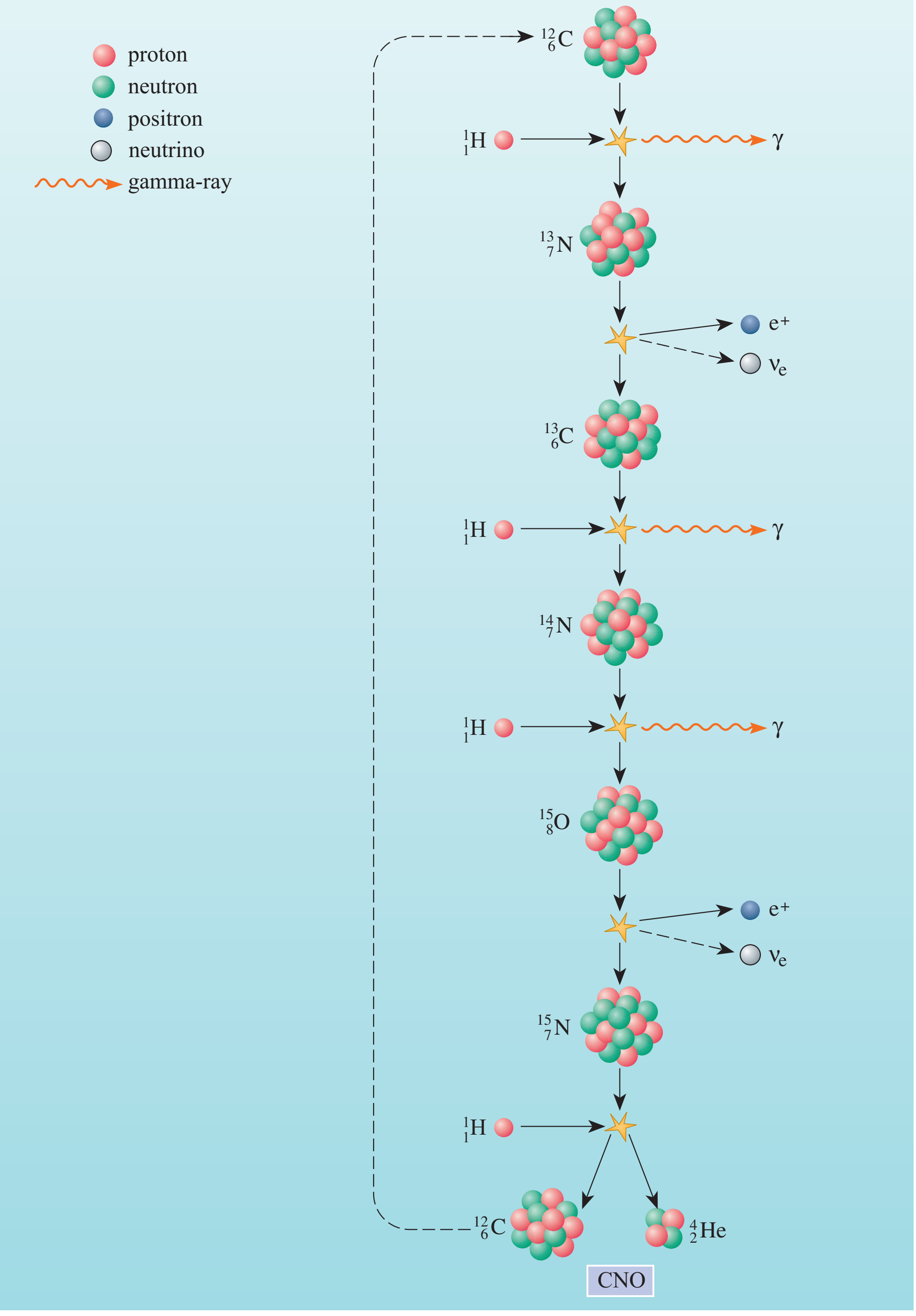


Figure 6.8 Details of each of the processes which convert hydrogen into helium in stars in the CNO cycle.

catalysts – that is, they help the reactions to take place. Although the relative abundances of the various isotopes of C, N and O may change, the combined concentration of these three elements, which is anyway low in main sequence stars, remains unchanged. This set of reactions is termed the **CNO cycle** and details are provided in Figure 6.8.

The net reactions for each of these processes are very similar:

ppI $4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\text{e}^+ + 2\nu_\text{e} + 2\gamma$

ppII $4\,{}^1_1\text{H} + \text{e}^- \rightarrow {}^4_2\text{He} + \text{e}^+ + 2\nu_\text{e} + 2\gamma$

ppIII $4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\text{e}^+ + 2\nu_\text{e} + 3\gamma$

CNO $4\,{}^1_1\text{H} \rightarrow {}^4_2\text{He} + 2\text{e}^+ + 2\nu_\text{e} + 3\gamma$

Figure 6.9 shows the rate of energy release as a function of temperature for both the pp and CNO reaction chains (the three pp chains have been added together for this purpose). As we can see, up to a temperature of about 18×10^6 K, it is the pp cycles that contribute most to energy generation within stars. Above that temperature, even though the pp cycles generate energy at an increasing rate, they are outstripped by the rate due to the CNO cycle. It is clear that the rate of energy generation depends heavily on the temperature – note the logarithmic scale. In the region where the two sets of reactions (pp and CNO) contribute equally to energy generation, the rates of energy production can be approximated by the following simple relations. For the pp chains

$$R_{\text{pp}} \propto n^2T^4 \tag{6.9}$$

and for the CNO cycle

$$R_{\text{CNO}} \propto n^2T^{17} \tag{6.10}$$

where R is the appropriate rate of energy generation per unit volume of gas, n is the number density of the reactant nuclei, and T is the temperature. The very high powers to which the temperature is raised in these equations go some way to explaining why certain stellar properties are so sensitive to a star's temperature. For example, in Section 4.2.4 you were told that for a 'mere' increase in mass along the main sequence of a factor of 500, the corresponding stellar luminosity increased by a factor of 10^{10}! This incredibly sensitive dependence of luminosity on mass can now be understood in the light of Equations 6.9 and 6.10, if you recall from Section 6.2.2 that the greater the mass of a star, the greater the temperature of its core.

The designation of *upper* and *lower* main sequence is a reflection of the division in mass (and therefore temperature) between those stars in which the pp chains dominate and those in which it is the CNO cycle that takes a dominant role. This

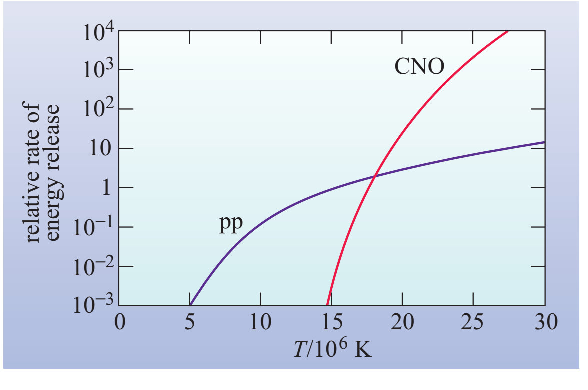


Figure 6.9 The rate of energy release for the three pp and CNO reaction chains as a function of temperature. A relative abundance of the elements as for the Sun has been assumed.

division occurs at a mass of around $1.5M_\odot$, corresponding to a peak core temperature of about 18×10^6 K. For more massive stars, in other words upper main sequence stars, the mass is greater and the temperature is higher, and the CNO cycle provides most of the energy generated. For less massive stars, lower main sequence stars, the temperature is lower and it is therefore the pp chains that are the main source of energy. The Sun falls into this latter category with a temperature nowhere higher than 16×10^6 K. In the Sun the ppI chain produces 85% of the energy, the ppII chain about 15% and the ppIII chain only 0.02%.

6.4 Stellar masses

6.4.1 Distribution of stellar masses

What can we say, if anything, about the *distribution* of the masses of stars? Are we equally as likely to find a low-mass star as to find a massive star?

- What technique is used for the direct determination of the masses of stars?
- The only *direct* method of measuring the mass of a star is by studying the dynamics of a binary system, through observations either of visual binaries or eclipsing spectroscopic binaries (see Section 3.3.7). For all other stars, the techniques are less direct.

Although the measurement of mass of any one star might not be possible to high precision, if we observe a large number of stars, we can expect the overall trend to be known to a reasonably high degree of accuracy. A plot of the relative numbers of stars of different masses, called the mass distribution function, is shown in Figure 6.10. This confirms what was stated in Sections 3.3.7 and 4.2.4 – namely that star masses cover the approximate range from $0.08M_\odot$ to $50M_\odot$. The figure also indicates that far more stars have a low mass than a large mass. While we can believe that the relative rarity of massive stars is a genuine feature, at the other end of the mass range, can we be sure that the observations that contribute to Figure 6.10 are not suffering from a selection effect? If we observe stars at random, there is clearly a bias towards observing brighter stars. The less massive a star is, the less luminous it is and the harder it is to observe, so that we might be underestimating the true number of very low mass stars. The mass distribution of stars is of particular significance to understanding the total mass and therefore gravitational stability of star clusters and galaxies. If the very faintest of stars are not easily visible, a large fraction of the mass of a star system may be due to undetected faint stars. Such cool stars will be more easily detectable (and identifiable) from observations made at infrared wavelengths (remember Wien's displacement law) and recent sky surveys have revealed that they are extremely common. Results from a survey entitled the Two Micron All Sky Survey, or 2MASS for short, have led to the definition of a new spectral class of stars (L-types), which are cooler than M-types (see Figure 6.11). They appear to be twice as numerous as all other stars, but because of their low masses, they probably make up only around 15% of the total mass of stars in the Galaxy.

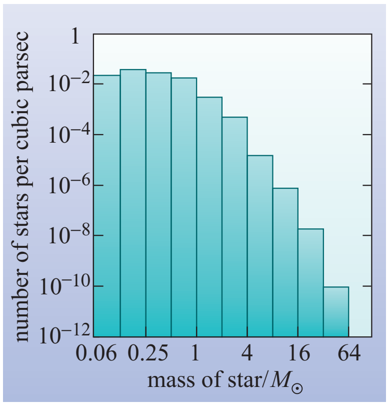


Figure 6.10 The number of stars observed as a function of stellar mass (based on observations of stars in the vicinity of the Sun). Note that both axes are logarithmic. In particular, each mass interval is twice the size of the preceding one. On a linear scale the dominance of the lowest mass stars would be even more pronounced.

If the mass distribution shown in Figure 6.10 continues to even smaller masses then we should have detected some extremely low-mass stars close to the Sun. Let's consider if there is a theoretical basis for defining a lower limit (and an upper limit) to the mass of a star.

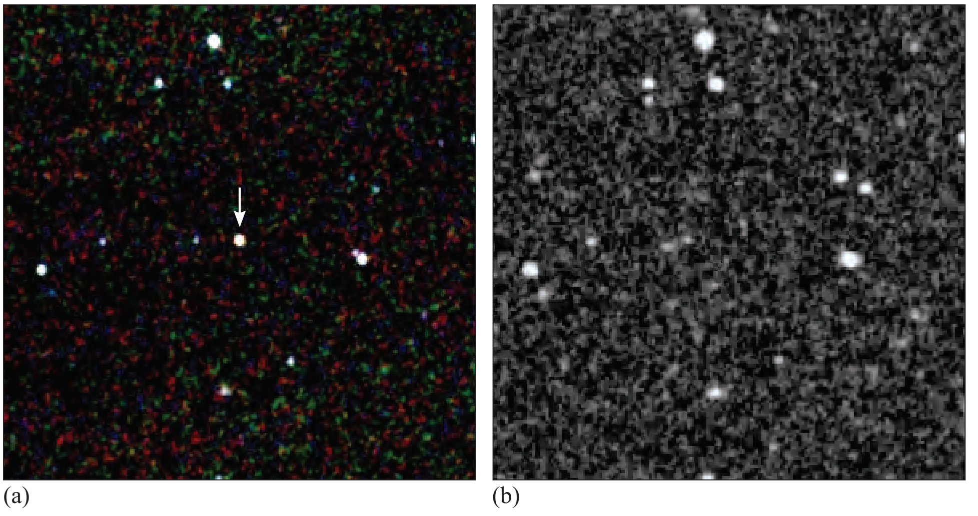


Figure 6.11 An L-type star discovered by the 2MASS sky survey. The left-hand image (a) shows the star (arrowed) as seen in the infrared 2MASS survey whereas it is so cool and red that it is not detected in the visible light image (b). (University of Massachusetts and Infrared Processing and Analysis Center/Caltech)

6.4.2 Brown dwarfs

We have already seen that the smaller the mass of a star, the lower the core temperature.

- What will be the effect on nuclear processes, such as the pp chains, of reducing the core temperature of a star?

- Figure 6.9 shows that the rate of energy generation will be reduced. Eventually, a temperature will be reached at which the nuclear reaction rate is insignificant.

Objects that are more massive than planets but do not have sufficient mass ($< 0.08M_\odot$) to run nuclear reactions at a rate high enough to match the surface radiation rate are referred to as **brown dwarfs**. This name is often taken to indicate their colour (at the predicted surface temperature of these objects, they would be redder than 'red dwarfs', i.e brown) and their size. However, *brown* was in fact used to imply *no* colour, i.e. beyond the red end of the visible spectrum (the more appropriate *black dwarf* was at the time in use to describe the end state of a more massive star). The likely evolutionary track of a brown dwarf on the H–R diagram is shown in Figure 6.12. It evolves straight past the main sequence because significant nuclear reactions are never triggered.

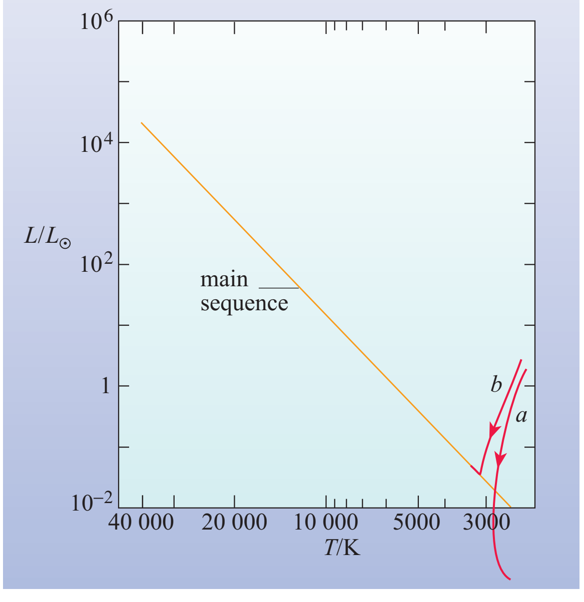
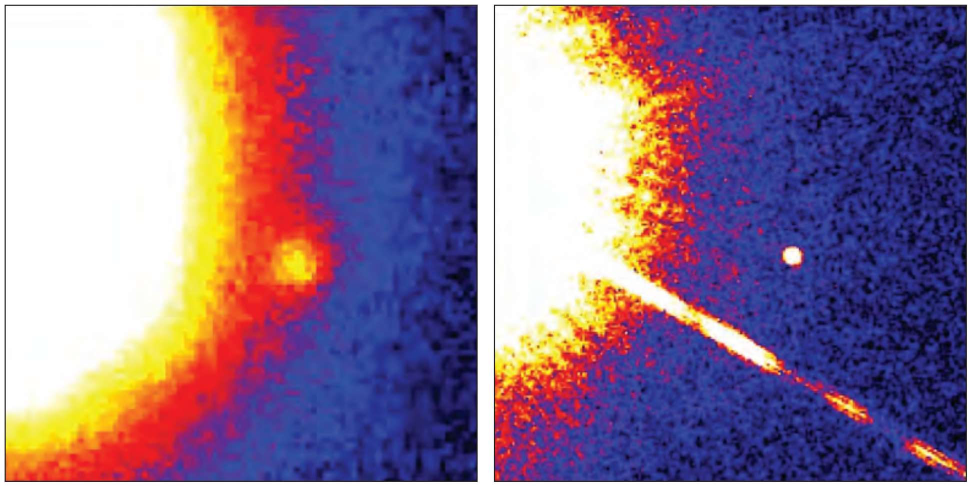


Figure 6.12 The path on the H–R diagram (plotted with the same axes as Figures 5.10 and 5.16), *a*, of a star of very low mass ($0.05M_\odot$), i.e. a brown dwarf, which evolves straight past the main sequence, and the path, *b*, of a star of mass $\approx 0.3M_\odot$, as it evolves on to the main sequence.

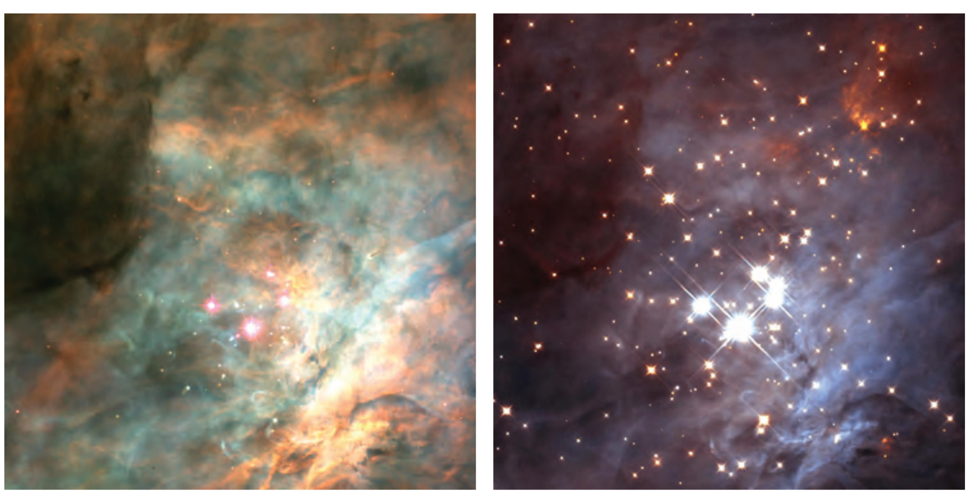
In addition to their low luminosity, their very low temperatures mean that they emit very little light at *visual* wavelengths. So any brown dwarf would therefore be very difficult to detect with certainty. Several brown dwarf candidates were 'discovered' in the 1990s, but there were difficulties associated with all of them, until 1994, when an object of mass between $0.03M_{\odot}$ and $0.05M_{\odot}$ was found orbiting the nearby star Gliese 229 (see Figure 6.13). Designated Gliese 229B, its surface temperature is around 1000 K and it is around 2000 times fainter than Gliese 229 which has spectral type M1 V, a temperature of 3800 K and is itself about 50 times less luminous than the Sun.



(a) (b)

Figure 6.13 Images of a brown dwarf orbiting at a distance of 44 AU (a similar distance to that of Pluto from our own Sun) from the star Gliese 229. (a) This image was taken in October 1994 from the Palomar Observatory, (b) is from the Hubble Space Telescope in November 1998. The spike running from the over-exposed image of Gliese 229 in the image (b) is an artifact caused by the telescope optics. ((a) T. Nakajima (Caltech)/ S. Durrance (JHU); (b) S. Kulkarni (Caltech), D. Golimowski (JHU)/ NASA)

What is the difference between brown dwarfs and planets like the gas giant planets Jupiter and Saturn, since in theory they both may have similar masses? One possible answer lies in their formation and composition. Brown dwarfs form directly out of the interstellar medium and will have the same composition as other stars (at the time of formation) whereas planets form by accretion of dust in nebulae surrounding protostars (although distinguishing between a planet and a brown dwarf in a binary system with an ordinary star – see Figure 6.13 – may not be possible). Another definition sometimes used is based on mass alone, with the division being at 0.013 solar masses or 13 Jupiter masses. This is the mass above which deuterium undergoes nuclear fusion, providing a low-level energy source.



(a) (b)

Figure 6.14 The central region of the Orion Nebula observed in visual light (a) and infrared (b). The four bright Trapezium stars that illuminate the nebula are clearly visible near the centre of both images. More than 300 stars are visible in the infrared image including about 50 brown dwarfs which are too faint and red to be seen in the visible image. ((a) NASA/K. L. Luhman (HSCA)/ G. Schneider, E. Young, G. Rieke, A. Cotera, H. Chen, M. Rieke, R. Thompson (Steward Observatory); (b) NASA/ C. R. O'Dell, S. K. Wong (Rice University))

Recent surveys using the Hubble Space Telescope have shown that brown dwarfs are indeed common (Figure 6.14) and that, like stars, their numbers increase with decreasing mass. They also appear to be more common in isolation than in orbit around other stars. Many of the L-type stars are likely to have masses below the limit for fusion of hydrogen and are therefore also brown dwarfs.

6.4.3 The most massive stars

What about the situation at the other end of the mass range?

- ■ Do you know of any reason why the mass of a star should have an upper limit?
- ❑ From the discussion so far, the only obvious limitation comes from the mass of the original contracting cloud. As typical clouds can have masses of several thousand solar masses or more, this is not a severe limitation. (However, you will see below that another limit exists.)

There is a process that is not important in most stars but which plays a crucial part in the most massive stars. You have already seen (Section 5.3.4) that electromagnetic radiation exerts a pressure called radiation pressure – the pressure exerted by photons of light, or of any other form of electromagnetic radiation. For photons within a black-body source (as is approximately true for typical stellar material), the radiation pressure is given by

$$P_{\text{rad}} = \tfrac{1}{3}\alpha T^4 \qquad (6.11)$$

where α is a constant, $7.55 \times 10^{-16}\,\text{N}\,\text{m}^{-2}\,\text{K}^{-4}$, and T is the temperature of the radiation source.

We can use a variant of Equation 6.3 to write the gas pressure as

$$P_{\text{gas}} = nkT \qquad (6.12)$$

where we have used the fact that the number density $n = N/V$.

QUESTION 6.6

Estimate the ratio of the radiation pressure to the gas pressure at the core of the Sun. (*Hint*: in order to calculate n, determine an average value by using the mass and radius of the Sun; also assume that the Sun is composed entirely of ionized hydrogen and that the core temperature is 1.6×10^7 K.) Now use Figure 2.1 for a more realistic estimate of the density at the centre of the Sun. How does this affect the answer? (A qualitative assessment will do.)

We see therefore that radiation pressure is almost negligible in comparison with the gas pressure, at least in a star like the Sun. However, is this likely to be the case for all stars?

- ■ What do you think will happen to the effect of radiation pressure in relation to the gas pressure as the temperature increases?
- ❑ In Question 6.6, it was shown that $P_{\text{rad}}/P_{\text{gas}} = \alpha T^3/3nk$. Therefore as T increases, the effect of radiation pressure compared with that of gas pressure will increase very quickly.

Whereas the stability of 'normal' main sequence stars results from a balance between the gravitational force and the force due to gas pressure, it appears that for more massive stars stability requires a balance between the gravitational force and the force due to radiation pressure. However, detailed modelling shows that radiation pressure increases so rapidly with temperature that such a star would be easily 'blown apart' by the radiation pressure. Detailed calculations show that the upper limit to the mass of a star is around $100M_{\odot}$ but its lifetime will be very short due to its rapid exhaustion of nuclear fuel.

6.4.4 Mass loss by stellar winds

Section 2.4.1 introduced us to the idea of the solar wind. It was suggested that it originates from material that has 'boiled off' from the solar corona, perhaps from regions where the magnetic field does not confine the material. If we make the assumption that the Sun is a typical main sequence star, then we might expect that a similar wind is exhibited by most stars – in which case, it should be termed a **stellar wind**.

The result of Question 2.10 shows that the rate of mass loss from a star like the Sun is small. Even if we sum this mass loss over the Sun's main sequence lifetime, it comes to little more than 10^{-4} of the total mass. Observations of the mass loss from other stars are very difficult owing to the low rates of mass loss and the distance to the stars, so data are very sparse. The rate of mass loss is expected to increase with increasing stellar mass such that for a massive star of $50M_{\odot}$, the mass loss will be of the order of $5 \times 10^{-7}M_{\odot}$ per year. In view of the very much shorter main sequence lifetime of such a star, the proportion of the total mass lost over the main sequence lifetime is still small.

Despite the relatively small amount of material involved in a typical stellar wind, it is one means by which material is returned from a star into the interstellar medium.

■ What do you expect the composition of this material to be?

❑ You should recall from Figure 6.5 that whether convection takes place in an envelope (for low-mass stars) or in a convective core (for higher mass stars), the products of the nuclear reactions in the core of a star are not brought to its surface by convection. The material being recycled by main sequence stellar winds is not therefore enriched in heavier elements (predominantly helium) from the nuclear processes occurring in the core, but is representative of the material from which the star first formed. (However, dust, comets and possibly even planets may fall into the parent star's atmosphere and enrich it in heavy elements.)

The main sequence stellar wind isn't the only means by which a star loses material to the interstellar medium. You have already encountered one other episode when significant mass loss takes place (the T Tauri phase, Section 5.3.4) and in Chapter 8 you will encounter others, including red giant stellar winds, planetary nebulae, novae and supernovae, which are many times more effective at removing mass than main sequence stellar winds.

6.5 Summary of Chapter 6

Stellar structure

- Stellar models allow astronomers to predict the variation of such physical properties as temperature, pressure and density within a star during its main sequence lifetime. These properties can be verified only indirectly.
- The internal temperatures of main sequence stars increase with increasing stellar mass.
- The mode of energy transfer within a star depends on its mass. For a star of mass less than about $1.5M_\odot$, convection is confined to an outer envelope. In a more massive star, the temperature gradient in the core is sufficient to allow convection to take place there. In such stars convection is confined to the core and the adjacent region.
- The main sequence on the H–R diagram represents the stable configuration of stars of different mass but similar composition, converting hydrogen into helium through nuclear fusion.
- Nuclear fusion is the source of energy that powers main sequence stars. No other mechanism is known that can provide the observed luminosities over the main sequence lifetime.
- The luminosity of main sequence stars is strongly dependent on their masses. The lifetime of a star on the main sequence decreases rapidly with increasing mass.

Nuclear reactions

- The detailed nuclear reactions that are responsible for converting hydrogen into helium depend on the core temperature, and therefore the mass of a star. For stars of mass less than about $1.5M_\odot$ (lower main sequence stars), the pp chains predominate. For more massive stars (upper main sequence stars), reactions involving carbon, nitrogen and oxygen as catalysts are dominant, and comprise the CNO cycle.

Limits to stellar masses

- Very low mass stars are much more common than those of high mass.
- Stars of mass less than $0.08M_\odot$ never achieve a sufficiently high core temperature to sustain hydrogen fusion and so become brown dwarfs.
- An upper limit of around $100M_\odot$ is found because more massive stars will not be stable against the force of radiation pressure.
- Stellar winds during the main sequence lifetime lead to a small proportion of a star's mass being ejected into the interstellar medium during this phase of evolution.

Questions

QUESTION 6.7

Outline the different means involved in the transport of energy from the centre of the Sun to the skin of a person standing on the surface of the Earth.

QUESTION 6.8

Assuming that the age of the Earth is 4.5×10^9 years, determine the mass of the most massive star now on the main sequence that was also on the main sequence at the time of the Earth's formation.

QUESTION 6.9

Discuss whether the fusion reaction

$$^{12}_{6}\mathrm{C} + {}^{12}_{6}\mathrm{C} \rightarrow {}^{24}_{12}\mathrm{Mg} + \gamma$$

(a) is possible and (b) could be an appreciable source of energy in main sequence stars. (At each of the mass numbers 12 and 24, these are the nuclides with the lowest rest energy per nuclide.)